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UTILITY PATENT APPLICATION TRANSMITTAL

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Attorney Docket No.	15-CT-5271
First Named Inventor or Application Identifier	Min Xie et al.
Title	METHOD AND APPARATUS FOR FAST NATURAL LOG(X) CALCULATION
Express Mail Label No	EL319728805US

APPLICATION ELEMENTS
See MPEP chapter 600 concerning utility patent application contents.

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JC530 U.S. PTO 09/507521 02/18/00

1. [X] Fee Transmittal Form *(e.g., PTO/SB/17)*
(Submit an original, and a duplicate for fee processing)
2. [X] Specification [*Total Pages* 17]
(Preferred arrangement set forth below)
   - Descriptive title of the Invention
   - Cross References to Related Applications
   - Statement Regarding Fed sponsored R & D
   - Reference to Microfiche Appendix
   - Background of the Invention
   - Brief Summary of the Invention
   - Brief Description of the Drawings *(if filed)*
   - Detailed Description
   - Claim(s)
   - Abstract of the Disclosure
3. [X] Drawing(s) *(35 USC 113)* [*Total Sheets* 2]
4. Oath or Declaration [*Total Pages*]
   a. [ ] Newly executed (original or copy)
   b. [ ] Copy from a prior application (37 CFR 1 63(d))
   (for continuation/divisional with Box 17 completed)
   [Note Box 5 below]
      i. [ ] DELETION OF INVENTOR(S)
      Signed statement attached deleting inventor(s) named in the prior application, see 37 CFR 1.63(d)(2) and 1.33(b).
5. [ ] Incorporation by Reference *(useable if Box 4b is checked)*
The entire disclosure of the prior application, from which a copy of the oath or declaration is supplied under Box 4b, is considered as being part of the disclosure of the accompanying application and is hereby incorporated by reference therein.

6. [ ] Microfiche Computer Program *(Appendix)*
7. Nucleotide and/or Amino Acid Sequence Submission
(if applicable, all necessary)
   a. [ ] Computer Readable Copy
   b. [ ] Paper Copy (identical to computer copy)
   c [ ] Statement verifying identity of above copies

ACCOMPANYING APPLICATION PARTS

8. [ ] Assignment Papers (cover sheet & document(s))
9. [ ] 37 CFR 3.73(b) Statement *(when there is an assignee)* [ ] Power of Attorney
10. [ ] English Translation Document *(if applicable)*
11. [ ] Information Disclosure Statement (IDS)/PTO-1449 [ ] Copies of IDS Citations
12. [ ] Preliminary Amendment
13. [X] Return Receipt Postcard (MPEP 503)
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14. [ ] Small Entity Statement(s) *(PTO/SB/09-12)* [ ] Statement filed in prior application, Status still proper and desired
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I certify that the attached continuation patent application of **MIN XIE AND SHARON X. WANG** for **METHOD AND APPARATUS FOR FAST NATURAL LOG*(X)* CALCULATION (Attorney Docket No.15-CT-5271)** including:

- Certificate of Mailing Via Express Mail (1 pg.)
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- Fee Transmittal (in duplicate) (1 pg.)
- Declaration and Power of Attorney (2 pgs.)(for purposes of identification only)
- Eight (8) pages of specification; eight (8) pages of claims; one (1) page of abstract
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METHOD AND APPARATUS FOR FAST NATURAL LOG(X) CALCULATION

BACKGROUND OF THE INVENTION

The invention relates generally to methods and apparatus for computing a computationally intensive algorithm, and more specifically to a method and apparatus for computing log(x), or equivalently, -log(x), in a manner that is particularly useful for computed tomographic image processing and other applications.

In at least one known computed tomography (CT) imaging system configuration, an x-ray source projects a fan-shaped beam which is collimated to lie within an X-Y plane of a Cartesian coordinate system and generally referred to as the "imaging plane." The x-ray beam passes through the object being imaged, such as a patient. The beam, after being attenuated by the object, impinges upon an array of radiation detectors. The intensity of the attenuated beam radiation received at the detector array is dependent upon the attenuation of the x-ray beam by the object. Each detector element of the array produces a separate electrical signal that is a measurement of the beam attenuation at the detector location. The attenuation measurements from all the detectors are acquired separately to produce a transmission profile.

In known third generation CT systems, the x-ray source and the detector array are rotated with a gantry within the imaging plane and around the object to be imaged so that the angle at which the x-ray beam intersects the object constantly changes. A group of x-ray attenuation measurements, i.e., projection data, from the detector array at one gantry angle is referred to as a "view." A "scan" of the object comprises a set of views made at a different gantry angles, or view angles, during one revolution of the x-ray source and detector. In an axial scan, the projection data is processed to construct an image that corresponds to a two dimensional slice taken through the object. One method for reconstructing an image from a set of projection

data is referred to in the art as the filtered back projection technique. This process converts the attenuation measurements from a scan into intergers called "CT numbers" or "Hounsfield units," which are used to control the brightness of a corresponding pixel on a cathode ray tube display.

The negative natural logarithm function -log(x) is an important yet computationally intensive algorithm in computed tomographic (CT) image processing. In known systems, a 5th order polynomial is used to approximate the function. However, this polynomial still consumes more than 20% of the total image processing time and generates a relatively large approximation error and error standard deviation.

A positive floating point number x can be represented by an expression written as:

$$x = m \times 2^{e} \qquad (1)$$

where m ($1 \le m < 2$) is a mantissa and e is a binary exponent.

Using equation (1), -log(x) can be written as:

$$y = -\log(x) = -\log(m) - e \times \log(2) \qquad (2)$$

The following equation uses a finite order polynomial to approximate log(m) in a region $1 \le m < 2$. Generally speaking, the higher the order of the polynomial, the better the approximation will be, but the computational load is in proportion to the order of the polynomial. For example, a 5th order polynomial presently used is written as:

$$\log(m) \approx (a_0 + a_1 m + a_2 m^2 + a_3 m^3 + a_4 m^4 + a_5 m^5) \qquad (3a)$$

or as:

$$y = -\log(x) \approx -(a_0 + m(a_1 + m(a_2 + m(a_3 + m(a_4 + a_5 m)))) + e \times a_6) \qquad (3b)$$

In equation (3b), $a_0 \sim a_6$ are precalculated constants. To compute -log(x), six additions and six multiplications are required, plus mantissa and exponent extractions.

To process images more efficiently and accurately, it would be desirable to provide methods and apparatus to reduce the complexity of the approximation used to calculate -log(x) while achieving numerical accuracy consistent with IEEE (Institute of Electrical and Electronic Engineers) floating-point precision.

BRIEF SUMMARY OF THE INVENTION

There is therefore provided, in one embodiment of the present invention, a method for computing a natural logarithm function that includes steps of: partitioning a mantissa region between 1 and 2 into N equally spaced sub-regions; precomputing centerpoints a_i of each of the N equally spaced sub-regions, where $i = 0, \ldots, N-1$; selecting N sufficiently large so that, for each sub-region, a first degree polynomial in m computes log(m) to within a preselected degree of accuracy for any m within the sub-region, where m is a mantissa of a binary floating point representation of a number; and computing a value of log(x) for a binary floating point representation of a particular number x stored in a memory of a computing device utilizing the first degree polynomial in m.

It will be seen that this embodiment and others described herein reduce the complexity of approximations used to calculate natural logarithms while achieving numerical accuracy consistent with IEEE floating point precision.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is a pictorial view of a CT imaging system.

Figure 2 is a block schematic diagram of the system illustrated in Figure 1.

Figure 3 is a representation of a number stored in IEEE single-precision binary floating point format, partitioned as in one embodiment of the invention.

DETAILED DESCRIPTION OF THE INVENTION

Referring to Figures 1 and 2, a computed tomography (CT) imaging system 10 is shown as including a gantry 12 representative of a "third generation" CT scanner. Gantry 12 has an x-ray source 14 that projects a beam of x-rays 16 toward a detector array 18 on the opposite side of gantry 12. Detector array 18 is formed by detector elements 20 which together sense the projected x-rays that pass through an object 22, for example a medical patient. Detector array 18 may be fabricated in a single slice or multi-slice configuration. Each detector element 20 produces an electrical signal that represents the intensity of an impinging x-ray beam and hence the attenuation of the beam as it passes through patient 22. During a scan to acquire x-ray projection data, gantry 12 and the components mounted thereon rotate about a center of rotation 24.

Rotation of gantry 12 and the operation of x-ray source 14 are governed by a control mechanism 26 of CT system 10. Control mechanism 26 includes an x-ray controller 28 that provides power and timing signals to x-ray source 14 and a gantry motor controller 30 that controls the rotational speed and position of gantry 12. A data acquisition system (DAS) 32 in control mechanism 26 samples analog data from detector elements 20 and converts the data to digital signals for subsequent processing. An image reconstructor 34 receives sampled and digitized x-ray data from DAS 32 and performs high speed image reconstruction. The reconstructed image is applied as an input to a computer 36 which stores the image in a mass storage device 38.

Computer 36 also receives commands and scanning parameters from an operator via console 40 that has a keyboard. An associated cathode ray tube display 42 allows the operator to observe the reconstructed image and other data from computer 36. The operator supplied commands and parameters are used by computer

36 to provide control signals and information to DAS 32, x-ray controller 28 and gantry motor controller 30. In addition, computer 36 operates a table motor controller 44 which controls a motorized table 46 to position patient 22 in gantry 12. Particularly, table 46 moves portions of patient 22 through gantry opening 48.

A negative natural logarithm function -log(m) is used by image reconstructor 34 to generate images. In one embodiment of the invention, the function log(m) of equation (3a) above is written as:

$$\log(m) = \log(a) + \frac{\log(m)'\big|_{m=a}}{1!}(m-a) + \frac{\log(m)''\big|_{m=a}}{2!}(m-a)^2 + \cdots + \frac{\log(m)^{(n)}\big|_{m=a}}{N!}(m-a)^n \tag{4a}$$

or as:

$$\log(m) \approx \log(a) + \frac{(m-a)}{a} - \frac{(m-a)^2}{2a^2} + \cdots \tag{4b}$$

where a is a known reference point. The error of the above function is written as:

$$error \le \left|\frac{\log(m)^{(n+1)}\big|_{m=a}}{N!}(m-a)^{n+1}\right| \tag{5}$$

Because (m-a)<1, there are two ways to minimize the error. One way is to increase the order of the approximation, and the other is to minimize the distance from m to a. Because mantissa m is between 1 and 2, in one embodiment of the present invention, the region between 1 and 2 is partitioned into N equally spaced sub-regions. Centers of each of the sub-regions are precomputed and used as reference points in equations (4a) and (4b). By partitioning into a sufficiently large number of sub-regions, a low order polynomial function produces sufficient accuracy for CT imaging purposes. In particular, by selecting a sufficiently large number of sub-regions, for any m within any particular sub-region, log(m) is computed by a first-

degree polynomial to within a preselected degree of accuracy within that sub-region. For example, computer 36 uses the first degree polynomial in m to compute values of $\log(x)$ for binary floating point representations of particular numbers x stored in its memory.

A $\log(m)$ approximation that is based on a first order polynomial with a set of precalculated reference points is written as follows:

$$\log(m) \approx \log(a_i) + \frac{(m - a_i)}{a_i}; \quad i = 0, \ldots, N-1; \quad 1 \le a_i < 2 \qquad (6)$$

where a_i is a closest reference point to a given mantissa m.

Rather than compute a sub-region index using $i = round((m-1) \times N)$, which would require six operations, one embodiment of the present invention reduces computation load as follows. A partitioning algorithm divides the mantissa of a binary floating point number in memory into two sub-regions. The sub-regions have index i and Δx, where Δx is a distance from mantissa m to reference point a_i. Indices i and Δx are directly extracted from an IEEE floating-point number stored in a computer system, thereby reducing computation time and improving accuracy. In one embodiment, mantissa partitioning occurs as illustrated in Figure 3, in which index i ranges from 0 to 127 and each region represents information extracted from the datum shown in Figure 3. More particularly, in a single precision IEEE floating point number, b_{31} represents a sign bit, b_{30} the most significant bit of exponent e, b_{23} the least significant bit of exponent e, b_{22} the most significant bit of mantissa m, and b_0 the least significant bit of mantissa m. (If it is desired to use a different designation for the numbering of bits b, those skilled in the art can make the appropriate changes required in the description for notational consistency.) In this single precision embodiment, exponent e is extracted directly from bits b_{30} to b_{23}; region i is extracted directly from bits b_{22} to b_{16}; and Δx (a distance from mantissa m to reference point a_i) is extracted directly from bits b_{15} to b_0.

Using the extraction illustrated in Figure 3, a maximum error of equation (6) in each sub-region is estimated by an expression written as:

$$error \leq \frac{1}{2a_i^2} \times \left(\frac{1}{2N}\right)^2; \quad i = 0,\ldots,N-1; \quad 1 \leq a_i < 2 \tag{7a}$$

From equation (7a), it is seen that the error of the first order approximation is always positive, so that the error is biased. To minimize the maximum error, in one embodiment the mean error of equation (7a) is subtracted from equation (6). Thus, the unbiased error is written as:

$$|error| \leq \left(\frac{1}{4Na_i}\right)^2; \quad i = 0,\ldots,N-1; \quad 1 \leq a_i < 2 \tag{7b}$$

Subtracting equation (7b) from equation (6) results in an unbiased first order polynomial function for -log(x) written as:

$$y = -\log(x) \approx b_i + c_i \Delta x + e \times \log(2) \tag{8}$$

for $i = 0,\ldots,N-1$

$$\begin{aligned} b_i &= -\log(a_i) + \left(\frac{1}{4a_i N}\right)^2 - \left(1 + \frac{1}{2N}\right)\frac{1}{a_i} \\ c_i &= -1/a_i \end{aligned} \tag{9}$$

where $a_i = 1 + \frac{i+0.5}{N}$, and Δx is a distance from mantissa m to reference point a_i. The value Δx is extracted directly from an IEEE floating point datum. In one embodiment, log(2) and the b_i and c_i are pre-calculated and saved in a look-up table at initialization time. In one embodiment, the values b_i are determined from precomputed values of log(a_i). For purposes of comparison, computation of equation (8) requires only 1/3 as much time as is required to calculate the 5th order approximation of equation (3b).

Taking into account the relationship of m and Δx, an approximation to $\log(x)$ (or equivalently $-\log(x)$) is computed using a polynomial of first degree in m and a precomputed value of $\log(a_i)$.

In one embodiment of the present invention, image reconstructor 34 is configured with software or firmware to compute logarithms using one or more of the methods of the present invention, when CT imaging system 10 images an object from acquired projection data.

From the preceding description of various embodiments of the present invention, it is evident that the complexity of the approximation used to calculate $-\log(x)$ is reduced, while numerical accuracy consistent with IEEE (Institute of Electrical and Electronic Engineers) floating-point precision is maintained. Thus, images processed by CT imaging system 10 are processed more efficiently, and without loss of detail. Although particular embodiments of the invention have been described and illustrated in detail, it is to be clearly understood that the same is intended by way of illustration and example only and is not to be taken by way of limitation. For example, embodiments of the improved computation for $\log(x)$ and $-\log(x)$ can be incorporated into any computational system requiring increased efficiency while maintaining computational accuracy. In addition, the present invention is suitable for use with floating point numbers having greater or lesser precision than those discussed in detail in this description. The modifications necessary to accommodate such different precisions will be apparent to those skilled in the art, once the invention described herein is thoroughly understood. Accordingly, the spirit and scope of the invention are to be limited only by the terms of the appended claims and legal equivalents.

WHAT IS CLAIMED IS:

1. A method for computing a natural logarithm function comprising the steps of:

partitioning a mantissa region between 1 and 2 into N equally spaced sub-regions;

precomputing centerpoints a_i of each of the N equally spaced sub-regions, where $i = 0, \ldots, N-1$;

selecting N sufficiently large so that, for each sub-region, a first degree polynomial in m computes $\log(m)$ to within a preselected degree of accuracy for any m within the sub-region, where m is a binary mantissa of a binary floating point representation of a number; and

computing a value of $\log(x)$ for a binary floating point representation of a particular number x stored in a memory of a computing device utilizing the first degree polynomial in m.

2. A method in accordance with Claim 1 wherein the particular number x has a binary exponent e in addition to the binary mantissa m;

and further wherein computing a value of $\log(x)$ for the binary floating point representation of the particular number x comprises the steps of:

partitioning a mantissa m of a binary representation of x in a memory, the representation of x including a binary exponent e and the binary mantissa m, wherein a first, most significant part of the partition corresponds to a region i and a second, less significant part of the partition corresponds to a region Δx, where Δx is a distance from mantissa m to reference point $a_i = 1 + \frac{i + 0.5}{N}$; and

computing an approximation to $\log(x)$, using a polynomial of first degree in m and a precomputed value of $\log(a_i)$.

3. A method in accordance with Claim 2 wherein computing the approximation to log(x) comprises the step of computing an approximation written as:

$$\log(m) \approx \log(a_i) + \frac{(m - a_i)}{a_i};$$

where a_i is a closest reference point to the binary mantissa m of the number x; and

$$1 \le a_i < 2.$$

4. A method in accordance with Claim 2 wherein computing an approximation to log(x) comprises the step of computing an approximation written as:

$$y = -\log(x) \approx b_i + c_i \Delta x + e \times \log(2)$$

for $i = 0, \ldots, N-1$

where:

$$b_i = -\log(a_i) + \left(\frac{1}{4a_i N}\right)^2 - \left(1 + \frac{1}{2N}\right)\frac{1}{a_i}; \text{and}$$

$$c_i = -1/a_i.$$

5. A method in accordance with Claim 4 further comprising the steps of precomputing a value for log(2), and, for each i, precomputing each value of b_i and c_i.

6. A method in accordance with Claim 5 further comprising the step of storing the precomputed values of b_i and c_i in a look-up table.

7. A method in accordance with Claim 2 wherein the number x is represented by a 32-bit representation having a sign bit, an 8-bit exponent, and a 23-bit binary mantissa m having bits b_{22} to b_0 in order of significance with b_{22} being a bit of greatest significance; and the step of partitioning the mantissa m comprises the

step of selecting a first group of bits b_{22} through b_{16} as index i and bits b_{15} through b_0 as Δx.

8. A method in accordance with Claim 1 utilized in a computed tomography (CT) scanner for generating an image of an object from acquired projection data of the object.

9. A method in accordance with Claim 8 wherein said natural logarithm is used in an image reconstructor to generate the image of the object.

10. A method in accordance with Claim 8 wherein the particular number x has a binary exponent e in addition to the binary mantissa m;

and further wherein computing a value of $\log(x)$ for the binary floating point representation of the particular number x comprises the steps of:

partitioning a mantissa m of a binary representation of x in a memory, the representation of x including a binary exponent e and the binary mantissa m, wherein a first, most significant part of the partition corresponds to a region i and a second, less significant part of the partition corresponds to a region Δx, where Δx is a distance from mantissa m to reference point $a_i = 1 + \frac{i+0.5}{N}$; and

computing an approximation to $\log(x)$, using a polynomial of first degree in m and a precomputed value of $\log(a_i)$.

11. A method in accordance with Claim 10 wherein computing the approximation to $\log(x)$ comprises the step of computing an approximation written as:

$$\log(m) \approx \log(a_i) + \frac{(m - a_i)}{a_i};$$

where a_i is a closest reference point to the mantissa m; and

$1 \le a_i < 2$.

12. A method in accordance with Claim 10 wherein computing an approximation to log(x) comprises the step of computing an approximation written as:

$$y = -\log(x) \approx b_i + c_i \Delta x + e \times \log(2)$$

for $i = 0, \ldots, N-1$

where:

$$b_i = -\log(a_i) + \left(\frac{1}{4a_i N}\right)^2 - \left(1 + \frac{1}{2N}\right)\frac{1}{a_i}; \text{ and}$$

$$c_i = -1/a_i.$$

13. A method in accordance with Claim 12 further comprising the steps of precomputing a value for log(2), and, for each i, precomputing each value of b_i and c_i.

14. A method in accordance with Claim 13 further comprising the step of storing the precomputed values of b_i and c_i in a look-up table.

15. A computing device comprising a memory in which binary floating point representations of particular numbers are stored, said device being configured to:

partition a mantissa region between 1 and 2 into N equally spaced sub-regions;

precompute centerpoints a_i of each of the N equally spaced sub-regions, where i=0,...,N-1, wherein N is sufficiently large so that, within each sub-region, a first degree polynomial in m computes log(m) to within a preselected degree of accuracy for any m within the sub-region, where m is a binary mantissa of a binary floating point representation of a number; and

compute a value of log(x) for a binary floating point representation of a particular number x stored in said memory utilizing the first degree polynomial in m.

16. A computing device in accordance with Claim 15 wherein the particular number x has a binary exponent e in addition to the binary mantissa m;

and wherein said device being configured to compute a value of log(x) for the binary floating point representation of the particular number x comprises said device being configured to:

partition a mantissa m of a binary representation of x in a memory of said device, the representation of x including a binary exponent e and the binary mantissa m, wherein a first, most significant part of the partition corresponds to a region i and a second, less significant part of the partition corresponds to a region Δx, where Δx is a distance from mantissa m to reference point $a_i = 1 + \frac{i+0.5}{N}$; and

compute an approximation to log(x), using a polynomial of first degree in m and a precomputed value of log(a_i).

17. A computing device in accordance with Claim 16 wherein said device being configured to compute the approximation to log(x) comprises said device being configured to compute an approximation written as:

$$\log(m) \approx \log(a_i) + \frac{(m - a_i)}{a_i};$$

where a_i is a closest reference point to the binary mantissa m of the number x; and

$$1 \le a_i < 2.$$

18. A computing device in accordance with Claim 16 wherein said device being configured to compute an approximation to log(x) comprises said device being configured to compute an approximation written as:

$$y = -\log(x) \approx b_i + c_i \Delta x + e \times \log(2)$$

for $i = 0,\ldots,N-1$

where:

$$b_i = -\log(a_i) + \left(\frac{1}{4a_iN}\right)^2 - \left(1+\frac{1}{2N}\right)\frac{1}{a_i}\text{; and}$$

$$c_i = -1/a_i.$$

19. A computing device in accordance with Claim 18 further configured to precompute a value for log(2), and, for each i, to precompute each value of b_i and c_i.

20. A computing device in accordance with Claim 19 further configured to store the precomputed values of b_i and c_i in a look-up table.

21. A computing device in accordance with Claim 16 wherein the number x is represented by a 32-bit representation having a sign bit, an 8-bit exponent, and a 23-bit binary mantissa m having bits b_{22} to b_0 in order of significance with b_{22} being a bit of greatest significance; and wherein said device being configured to partition the mantissa m comprises said device being configured to select a first group of bits b_{22} through b_{16} as index i and bits b_{15} through b_0 as Δx.

22. A computing device in accordance with Claim 15 in a computed tomography (CT) scanner and utilized by said CT scanner for calculating logarithms when said CT scanner generates an image of an object from acquired projection data of the object.

23. A computing device in accordance with Claim 22 wherein said CT scanner utilizes said computing device to calculate natural logarithm in an image reconstructor to generate the image of the object.

24. A computing device in accordance with Claim 22 wherein the particular number x is stored with a binary exponent e in addition to the binary mantissa m;

and further wherein said device being configured to compute a value of log(x) for the binary floating point representation of the particular number x comprises said device being configured to:

partition a mantissa m of a binary representation of x in a memory, the representation of x including a binary exponent e and the binary mantissa m, wherein a first, most significant part of the partition corresponds to a region i and a second, less significant part of the partition corresponds to a region Δx, where Δx is a distance from mantissa m to reference point $a_i = 1 + \frac{i+0.5}{N}$; and

compute an approximation to log(x), using a polynomial of first degree in m and a precomputed value of log(a_i).

25. A computing device in accordance with Claim 24 wherein said device being configured to compute the approximation to log(x) comprises said device being configured to compute an approximation written as:

$$\log(m) \approx \log(a_i) + \frac{(m - a_i)}{a_i};$$

where a_i is a closest reference point to the mantissa m; and

$1 \leq a_i < 2.$

26. A computing device in accordance with Claim 24 wherein said device being configured to compute an approximation to log(x) comprises said device being configured to compute an approximation written as:

$$y = -\log(x) \approx b_i + c_i \Delta x + e \times \log(2)$$

for $i = 0, \ldots, N-1$

where:

$$b_i = -\log(a_i) + \left(\frac{1}{4a_i N}\right)^2 - \left(1 + \frac{1}{2N}\right)\frac{1}{a_i}; \text{ and}$$

$$c_i = -1/a_i.$$

27. A computing device in accordance with Claim 26 further configured to precompute a value for log(2), and, for each i, to precompute each value of b_i and c_i.

28. A computing device in accordance with Claim 27 further configured to store the precomputed values of b_i and c_i in a look-up table.

METHOD AND APPARATUS FOR FAST NATURAL LOG(*X*) CALCULATION

ABSTRACT OF THE DISCLOSURE

The present invention is, in one embodiment, a method for computing a natural logarithm function that includes steps of: partitioning a mantissa region between 1 and 2 into N equally spaced sub-regions; precomputing centerpoints a_i, of each of the N equally spaced sub-regions, where $i=0,\ldots,N-1$; selecting N sufficiently large so that, within each sub-region, a first degree polynomial in m computes $\log(m)$ to within a preselected degree of accuracy for any m within the sub-region, where m is a mantissa of a binary floating point representation of a number; and computing a value of $\log(x)$ for a binary floating point representation of a particular number x stored in a memory of a computing device utilizing the first degree polynomial in m.

This embodiment of the present invention and others described herein reduce the complexity of approximations used to calculate natural logarithms while achieving numerical accuracy consistent with IEEE floating point precision.

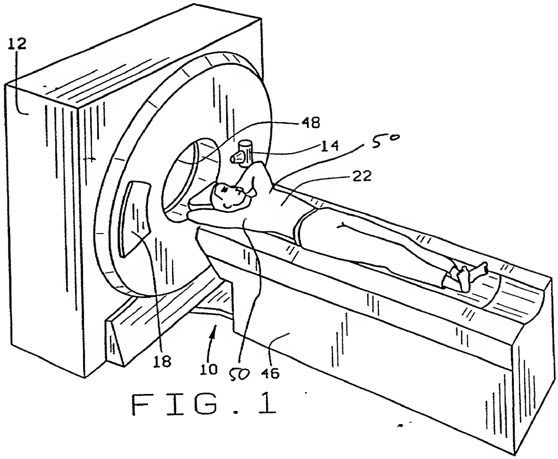
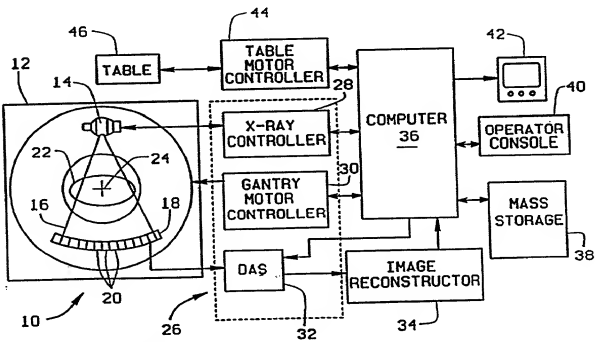

12
48
14
50
22
18
10
50
46

FIG. 1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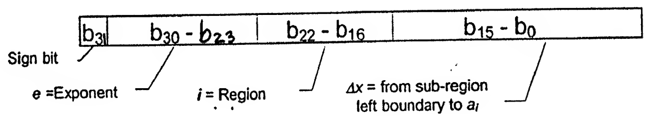
46
TABLE
TABLE MOTOR CONTROLLER
42
12
14
28
40
X-RAY CONTROLLER
COMPUTER 36
OPERATOR CONSOLE
22
24
30
GANTRY MOTOR CONTROLLER
16
18
MASS STORAGE
38
DAS
IMAGE RECONSTRUCTOR
20
10
26
32
34

FIG. 2



b_{31}
b_{30} - b_{23}
b_{22} - b_{16}
b_{15} - b_0
Sign bit
e =Exponent
i = Region
Δx = from sub-region
left boundary to a_i

Fig. 3

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DECLARATION AND POWER OF ATTORNEY	Attorney's Docket No. 15-CT-5271

As a below named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name.

I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled: **METHOD AND APPARATUS FOR FAST NATURAL LOG*(X)* CALCULATION**, the specification of which:

(check one) [X] is attached hereto

[ ] was filed on _________ as Application Serial No. _________,
and was amended on _________________.

I hereby state that I have reviewed and understand the contents of the above identified specification, including the claims, as amended by any amendment referred to above.

I acknowledge the duty to disclose information which is material to the examination of this application in accordance with Title 37, Code of Federal Regulations §1.56(a).

I hereby claim priority benefits under Title 35, United States Code, §120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, §112. I acknowledge the duty to disclose material information as defined in Title 37, Code of Federal Regulations, §1.56(a) which occurred between the filing date of the prior application and the national or PCT international filing date of this application:

Application Serial No.	Filing Date	Status (patented , pending, abandoned)

I hereby claim the benefit under Title 35, United States Code §119(e) of any United States provisional application(s) listed below:

Application Serial No.	Filing Date

________ Additional provisional application numbers are listed on a supplemental priority sheet attached hereto.

POWER OF ATTORNEY: As a named inventor, I hereby appoint the following attorney(s) and/or agent(s) to prosecute this application and transact all business in the Patent and Trademark Office connected therewith. (*list name and registration number)*

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I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application and any patent issued thereon.

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